Pergamon

Int. J. Solids Structures Vol. 33, No. 24, pp. 3539–3548, 1996

0020-7683/96 $15.00 + .00

0020–7683(95)00194–8

BUCKLING EXPERIMENTS ON DAMAGED CYLINDRICAL SHELLS

E. T. HAMBLY* and C. R. CALLADINE
Department of Engineering, University of Cambridge, Cambridge CB2 1PZ, U.K.

(*Received* 22 *February* 1995; *in revised form* 8 *August* 1995)

Abstract—It is well known that the buckling load of a thin cylindrical shell under axial compression depends significantly on the initial imperfection of the specimen. In this paper we focus attention on the effect of imperfections in the form of a particular kind of *damage* on the load-carrying capacity of a thin cylindrical shell. The study is an experimental one. The specimens used were steel drinks cans having a radius/thickness ratio of about 350. The damage was introduced by pressing in an indentation by hand and then pushing it out again: this left two locally damaged regions in the shell; and the level of damage could be characterised by the width of the initial indentation. The specimens were tested under eccentric axial compression. Careful visual observations were made of the behaviour of the shells in the damaged regions. In general, as the load increased a small dimple near the damaged zone grew in size, and turned gradually into a diamond shape. When the width of such a dent reached 25 mm, the can collapsed catastrophically. The nominal stress level in the region of the dent at buckling was around 0.24 of the classical buckling stress, whereas the buckling stress level for undamaged cans was 0.5 ± 0.1 of the classical value. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Thin cylindrical shells that sustain axial compressive loads are found in a very wide range of engineering applications; and engineers need to have a method for designing them safely, particularly in relation to buckling. The problem of understanding the buckling behaviour of thin shells in compression is a very old one; and we shall sketch the history of the various investigations that have been made over the years in order to set the stage for our own study.

Early investigators of the elastic buckling of cylindrical shells found theoretical formulae for the buckling load under axial compression without very much difficulty: the analysis ran along the lines of the standard Euler theory of buckling for an initially straight elastic column, and the aim of the investigations was to find the special load at which a non-symmetrical buckling mode could be in statical equilibrium—or what would now be called the point of bifurcation of equilibrium paths [see, e.g. Timoshenko (1953), Timoshenko and Gere (1961), Flügge (1973)]. That kind of theory was regarded as satisfactory until experiments showed that real shells usually buckled at much smaller loads than those indicated by what now had to be called the "classical" eigenvalue theory. It was soon appreciated that *non-linearity* was an important aspect of the actual behaviour, in contrast to the *linearised* behaviour as analysed in the classical theory [see Donnell (1934), Kármán *et al.* (1940)]. Also, the striking role of initial imperfections in reducing the load-carrying capacity of thin shells was discovered. The work of Koiter (1945, 1963, 1978) provided a major breakthrough in this field. He produced a "manageable" non-linear theory which was successful in linking the presence of small geometric imperfections with (i) maximum supportable loads that were low in relation to those of the classical theory; (ii) unstable post-buckling behaviour; and (iii) an expectation of wide scatter of measured loads in any real testing programme. In recent years Arbocz and his co-workers (1969, 1974, 1991) have made significant progress in predicting the buckling loads of real cylindrical shells from measurements of the geometric imperfections of these shells as manufactured. An important feature of the work has been the analysis of measured imperfections into harmonic component modes of initial geometric imperfection: such modes were a strong feature of

* Present address: Robert Benaim and Associates, East Fleet House, 55 Grafton Road, London NW5 3EL, U.K.

Koiter's scheme of analysis and explanation of the imperfection-sensitive buckling of thin-walled shells.

It is against this background that we shall investigate, by means of some simple experiments, the effect on the buckling of a cylindrical shell of a particular kind of imperfection for which, *prima facie*, a decomposition of the initial form of the shell into harmonic components is hardly relevant. We shall be mainly concerned with situations in which a cylindrical shell has been *damaged*, perhaps during its construction or operation, by being subjected to some sort of impact. We envisage here an event which produces a severe indentation of the shell, and which is followed by an attempt to restore the shell to its original form by "pushing out" the indentation, somehow or other.

Now when a relatively *thick*-walled, steel cylindrical shell is damaged by an impact, a relatively smooth-sided dent may be formed: we are thinking here of a high-pressure gas pipeline made from structural steel, with a radius-to-thickness ratio of around 40, that has been damaged by a falling bulldozer in the process of trench-filling [see Lancaster and Palmer (1992)]. In such a dented region the shell wall has obviously experienced both bending and stretching deformations, and those well into the plastic range. For *thin*-walled shells, however, the situation is rather different. Readers will no doubt be familiar with the behaviour of an empty drinks can (typically having a radius-to-thickness ratio of 350) when it is held in the hand and pushed in on one side by the thumb. In this case, the deformation has a strongly "inextensional" character, in the sense that if the indentation is pushed out, the can returns to much the same shape as before. There is little sign of widespread surface straining, and yet highly localised regions of damage are evident.

A particular feature of the geometry of the indentation produced by pressing the thumb hard into a drinks can is that the indented zone has a major feature in the form of a "crease" which lies in a plane perpendicular to the axis of the can, as shown schematically in Fig. 1(a). At each end of such a crease there is an awkward feature that evidently contains some highly localised compressive deformation in the axial direction, which is apparently required to make the edges of the indentation geometrically compatible with the distorted shape of the remainder of the can. These regions involve some small-scale "crinkling" of the surface; and when the crease is sufficiently wide—greater than, say, the radius of the can—these crinkles evidently involve localised plastic folding back-and-forth about a locally circumferential axis. When the can is restored again to its original shape overall, it is these localised regions of crinkling-damage that remain, as shown schematically in Fig. 1(b). (There would be no such damage if the deformation were truly inextensional. As Taylor (1942) has shown, the deformed surface can consist only of developable surfaces—planes,

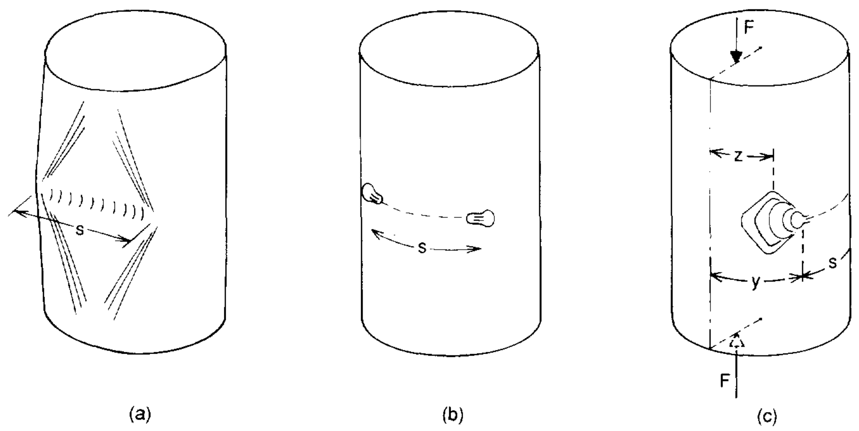


Fig. 1. (a) An empty drinks-can with a thumb-pressed indentation of width s. (b) The indentation of (a) has been pressed out, leaving behind damage in the form of two "dings"—shown schematically here by double-lines representing localised creases and small dimples or depressions beyond them. The separation s is measured between the ends of the two "creases". (c) The can has been rotated, and an eccentric axial load F is now applied, so that the nearer "ding" is distance y from the principal generator (marked with a chain-dotted line). As the load increases, the dimple grows and becomes a diamond-shaped dent. Just before the dent becomes unstable, its centre is a distance z from the principal generator.

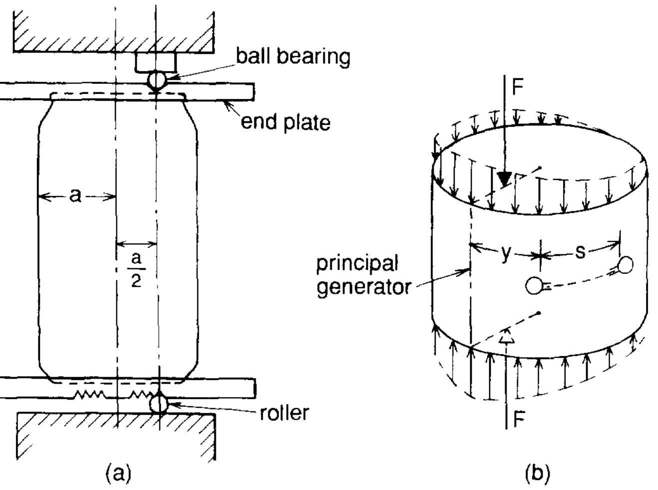


Fig. 2. (a) Schematic view of testing arrangement, showing the empty drinks-can between end plates and loaded at an eccentricity $a/2$ between a roller and a ball-bearing. The load is applied between the cross-head and platform of a Howden universal testing machine type EU 500. (b) Schematic view of a portion of the cylindrical shell showing, as bold arrows, the resultant compressive load; and as light arrows the nominal distribution of axial compressive load around the circumference. The principal generator is the most highly loaded line; and the distance y around the circumference to the nearer "ding" is measured from it.

cones and cylinders—if it is to be inextensional. The indented drinks can has portions of its surface that are plainly doubly-curved; and so there is some local crinkling.)

This well-known kind of deformation of thin cylindrical drinks cans gave us the idea for doing some simple experiments on the effect of imperfections in the form of local damage on the buckling of shells in compression. The supply of specimens would be plentiful. We would focus attention on the small regions of damage at the ends of the crease in the indentation; and we would use the width of the crease as a simple index of the severity of the damage.

In this paper we shall describe some experimental investigations of this kind, and our attempts to understand the process of buckling that is initiated by the presence of localised damage of this sort. Our work is in the nature of a pilot study; but nevertheless we have been able to draw some conclusions which are (we believe) both firm and interesting.

2. SET-UP FOR TESTING THE CYLINDRICAL SHELLS

In many experimental investigations into the buckling strength of cylindrical shells, great care is taken to ensure that the compressive loading is uniform around the circumference. To this end it is often arranged for the resultant compressive force to lie accurately along the axis of the cylinder. It is obvious from the description above that our particular procedure for introducing damage into a shell always produces two distinct regions of local damage. Thus, if we were to test the shell under a centrally aligned compressive load, these two regions of damage might well compete with each other as the focus of the buckling events that will lead, eventually, to the collapse of the shell. For this reason we decided deliberately to load the shell eccentrically, in order to produce a non-uniform compressive loading around the circumference. Our aim was to locate one particular small zone of damage in a region of the shell that was under relatively high axial compression, so that we could be confident in advance where the buckling process would be initiated.

Our arrangement for loading the empty drinks cans is shown schematically in Fig. 2(a). The two ends of the can are held in brass plates which contain machined recesses that fit precisely the form of the ends of the can. The obverse face of one plate contains a series of parallel V-grooves, so that the load can be applied through a steel rod or roller resting on a hard surface. The other plate contains a series of conical holes lying on a radius, so that the load can be applied through a ball-bearing. As indicated in Fig. 2(a), the load is applied to the specimen through the parallel platens of a universal testing machine that is capable of measuring the applied force and the displacement of the cross-head of the machine.

In a given test the load is applied on a line parallel to the axis of the specimen, by choosing the groove in the lower plate and the conical depression in the upper plate that are at equal distances from the centre. (It also has to be arranged, of course, that the hole in the upper plate lies in the central plane perpendicular to the grooves of the lower plate, as shown in Fig. 2(a).)

Another possible arrangement would be to have a set of grooves on both plates. This, however, would ensure that the two plates were obliged to rotate relative to each other about an axis parallel to the axes of the grooves. By applying the load on one plate through a ball-bearing, the relative rotation of the two plates is not so strongly inhibited: the arrangement ensures that the pattern of buckling is less constrained by the support-conditions of the specimen.

Although the end plates were machined in such a way that the load-axis could be varied in its position, we decided after some preliminary tests always to apply the load at a distance of $a/2$ from the axis of the cylinder, where a is the (mean) radius of the cylinder. According to the classical theory of compression of tubular members, such a load position provides a distribution of compressive stress around the circumference that varies from zero on one generator of the shell to twice the mean value on the generator diametrically opposite. This is shown schematically in Fig. 2(b): the variation of compressive stress around the circumference is proportional to $(1+\cos\theta)$, where θ is the circumferential angle (not shown in the diagram), measured from what we shall call the *principal generator*.

The deduction of this particular distribution of stress in the wall of the cylinder from the position of the applied load involves, of course, a number of standard assumptions in the classical "engineers' bending theory" [see, e.g. Timoshenko (1953) § 32]; and these are not necessarily warranted in short cylindrical specimens loaded in the manner of Fig. 2(a), and subjected to buckling. For this reason we shall regard the stress pattern shown in Fig. 2(b) as the "nominal" distribution of stress. It would, of course, be possible in principle to assay the distribution of stress in the shell as a particular test proceeded by means of strain gauges attached to the surface. However, for the sake of simplicity such an arrangement was not used in the present study.

All of the drinks cans tested were made of steel, and were of the same kind. The leading dimensions were as follows (see Fig. 2(a)):

$$\text{radius} = 33\,\text{mm}; \quad \text{thickness} = 0.095\,\text{mm}; \quad \text{length of cylindrical portion} = 95\,\text{mm}. \tag{1}$$

The thicknesses of the cans was measured as follows. A few cans were cut up with the aid of heavy scissors, and the cylindrical portions of them were unrolled into rectangles. Then a solvent was used to remove the brand-name paint on the outside, and the lacquer on the inside. The thickness was measured by means of a micrometer at numerous places over the surface of each rectangle. The thickness was constant to within about 1% over most of the cylindrical surface, but it increased by about 10% over about 1 cm at each end. Thus, for the purpose of our tests, the thickness value given above should be satisfactory. Some drinks manufacturers use both steel and aluminium cans; but in our study we used only the steel variety.

3. DELIBERATE DAMAGE: PATTERN AND SEVERITY

As already indicated, we damaged cans in a standard way by pressing them in by hand, as shown in Fig. 1(a). A little practice enables one to make an indentation of a particular width s; and we were thus able to classify the severity of damage according to this width. After some trial, we settled on the following series of particular widths for the indentation:

$$s = 30, 35, 39, 44, 50 \text{ mm} \qquad (2)$$

Indentations less than 20 mm wide did not seem to do any damage. If the indentation was less than about 32 mm wide—i.e. a little less than the radius of the can—it would spring back as soon as the pressure from the thumb was removed. Indentations of greater than about 32 mm in width did not restore themselves, but had to be assisted. Pinching in the area surrounding the indentation usually restored the shell to its original shape, more or less (as the reader can readily verify); but indentations of larger width tended to be more difficult to remove cleanly.

The indentations that were imparted to the cans in this investigation, of width between 30 and 50 mm, left behind a characteristic localised damage pattern, as already mentioned. We have coined the word "ding" to describe the locally-deformed feature that remains after an indentation has been pressed in and then removed. This feature includes a short, plastically deformed crease in line with the opposite ding, as shown schematically in Fig. 1(b); and this crease is surrounded by a slight depression. We use the symbol s to denote the separation of the two "dings"; this is taken as being equal to the width of the indentation.

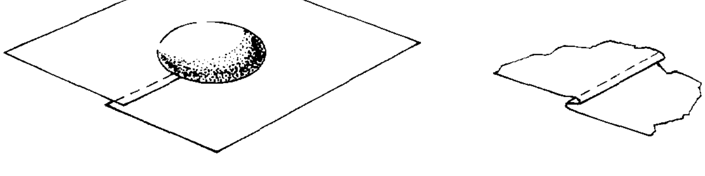
When $s > 40$ mm, close examination reveals not only the region of localised plastic creasing and depression, mentioned above, but also that the depression takes the form mainly of a tiny spherical *dimple* just beyond the end of the creased region. This arrangement is also sketched in Fig. 1(b). The diameter of the small dimple is usually about 6 mm. It is not difficult to understand how such a dimple can be formed. Thus, if one takes a piece of writing paper and cuts a slit in from one edge, and then makes the edges of the slit overlap each other uniformly by, say, 1 mm, then a dimple of much the same general kind is formed, as shown schematically in Fig. 3(a). The overlapping edges of the paper model, in a simple way, the creased material of the "ding", shown schematically in Fig. 3(b). The crease in the shell tends to be more severe at its outer end, and the associated spherical dimple is well-defined. A less-well defined depression may sometimes be detected at the inner edge of the crease. As we shall see, the dimple associated with a "ding" can play an important part in the buckling behaviour of the shell.

When a shell has been dented, as in Fig. 1(a), the two ends of the can are no longer plane: they "dip" a little on the dented side, as indicated schematically in the sketch. But after the dent has been removed, as in Fig. 1(b), the ends return to their original plane conformation; and in particular they fit the machined end-plates perfectly.

4. OBSERVATIONS

In general, the testing of specimens followed a set pattern. First, two "ding" imperfections were introduced to the shell as described above. Then the specimen was tested in compression until it buckled; and a load-displacement plot was produced. Throughout the test a written account was made of progressive changes in the appearance of the surface of the can.

The position of the "dings" with respect to the principal generator could be varied at will; and Figs 1(c) and 2(b) show the definition of a parameter y used for this classification.



(a) (b)

Fig. 3. (a) Schematic view of a dimple raised in a sheet of paper by making a cut and then overlapping the edges by a small, uniform amount. In practice, of course, the dimple is not as uniform as that shown here. (b) Schematic view of highly localised folding of the shell wall at a "ding": the adjoining dimple is not shown.

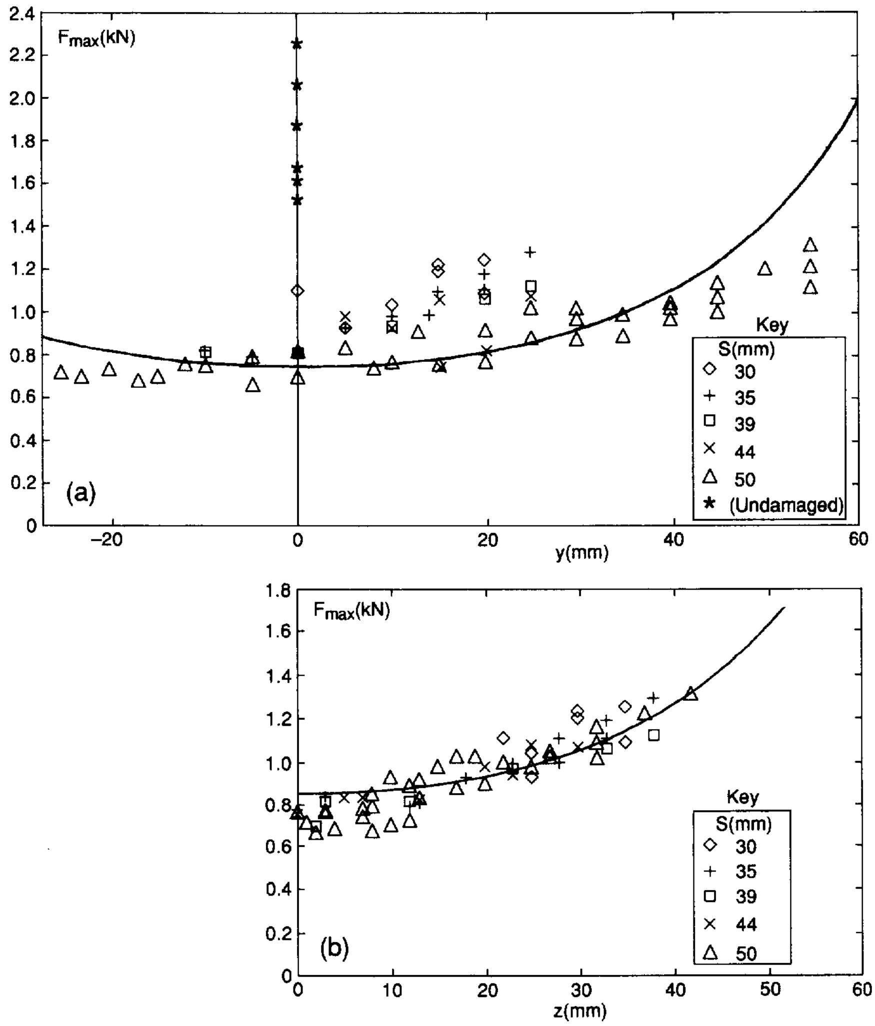


Fig. 4. (a) Plot of measured collapse load F_{max} against eccentricity y of "ding": see Figs 2(b) and 1(c). When the value of y is negative, the principal generator passes between the two "dings". The curve has equation $F_{max} = 1.5/(1+\cos(y/a))$. (b) Plot of F_{max} against eccentricity z of the centre of the diamond-shaped dent at the point of collapse: see Fig. 1(c). The curve has equation $F_{max} = 1.7/(1+\cos(z/a))$.

For positive values of y (as shown here) the principal generator lies to one side of the pair of "dings". In general the "ding" closer to the principal generator, being in a region of higher nominal compressive stress, becomes the focus of buckling. One possible experimental strategy would have been to set $y = 0$ for all tests, i.e. always to have the principal generator passing through one of the two "dings". As we shall see, however, there are advantages in using a range of values of y.

In summary, then, each test is characterized by the values of s, the separation of the "dings" and y, the distance of the more heavily stressed "ding" from the principal axis.

Tests were also performed on six undamaged specimens, so that a comparison could be made between the behaviour of damaged and undamaged shells. The load axis was eccentric by $a/2$ in these tests also. One reason for using an eccentric load was that most cans "as received" had detectable—though not severe—imperfections over parts of their surface; and the eccentric-loading arrangement made it possible to arrange for the principal generator to pass through a relatively unblemished portion of the shell. Undamaged shells were considerably stronger than the damaged ones, and they always failed in a sudden and catastrophic fashion: there was virtually no warning of impending collapse. After collapse the load-carrying capacity was much reduced. Values of the load at which buckling occurred are shown as fine-pointed stars, on the axis $y = 0$ in the plot of Fig. 4(a). (In testing undamaged shells in the manner of Fig. 2(a), it was found that the ends of the can tended to deform in an "elephant's foot" mode. This was rectified by embedding approximately 10 mm at each end of these specimens in epoxy resin. This precaution was found not to be needed in the case of the damaged shells.)

In contrast, the buckling failure of the 65 deliberately damaged shells was much less sudden and catastrophic. We shall describe first the behaviour under test of specimens ($s > 40$ mm) that have visible dimples, as in Fig. 1(b), adjacent to the "dings", and are set up with $y > 0$. In general, what happened as the load increased was that, at a certain stage—usually when about 2/3 of the eventual failure load had been reached—the small dimple close to the "ding" began to enlarge towards the principal generator. This enlargement always occurred about the fixed point where the plastically creased "ding" joined the dimple: the situation is shown schematically in Fig. 1(c). As already mentioned, the initial diameter of the dimple was usually about 6 mm, and as the load increased, the dimple grew larger. At first, the enlarged dimple retained its original circular shape; but as it grew further it began to change into more of a diamond-shaped dent, as shown.

When the circumferential width of the dent had reached about 20 mm, the diamond shape was well-defined, with a horizontal crease resembling to some extent the crease of the initial thumb-indentation. The diamond shapes always had almost equal width and height; that is, they were approximately square. Throughout this enlargement of the dimple into a diamond-shaped dent, the load continued to increase. Then, when the width of the diamond reached 25 mm, failure occurred suddenly; the shell crumpled, and the load-carrying capacity fell sharply. Thus, although the final collapse was similar in its general characteristics to that of an undamaged shell, the damaged shells all gave ample warning of impending failure through the steady growth of the dimple, and its change into a diamond-shaped dent.

The above description applies well to the specimens with dimples, except those that were tested with the principal generator passing between the "dings", i.e. $y < 0$: see below. In particular, the width of the diamond-shaped dent at the point of buckling was always equal to 25 mm—to within 1 or 2 mm—for the damaged specimens.

In specimens without dimples ($s \leqslant 35$ mm), however, the sequence of events before the maximum load was reached was somewhat different. In such cases the main focus of deformation was the "ding" itself. A new dimple formed out of the "ding", and then it grew in the manner just described, enlarging with one corner centred on the end of the crease and always growing *towards* the other "ding" and across the original indentation. In particular the dimple grew into a diamond-shaped dent; and again the dent became unstable when it grew to a width of 25 mm.

Lastly, we mention the specimens with dimples (i.e. the specimens having $s > 40$ mm) that were tested with $y < 0$. In these shells, as the applied load increased, a dimple formed out of the small depression mentioned at the end of Section 3 (see Fig. 1(b)), and grew in preference to the existing dimple. Just as in the case of specimens with $s \leqslant 35$ mm, as described above, the dimple grew towards the second "ding", becoming more diamond-shaped and again failing when it became 25 mm wide.

5. QUANTITATIVE OBSERVATIONS

Figure 4(a) shows an overall plot of the maximum load, F_{max}, sustained by the specimens against y, the distance of the closer "ding" from the principal generator, with the points for specimens having different values of s distinguished. Also marked, on the axis $y = 0$, are points corresponding to the testing of six undamaged shells, as already mentioned.

In this plot, the axial load is given in absolute terms. As shown in the Appendix, the axial force required to produce the "classical" buckling stress level at the principal generator is 3.6 kN for our specimens. Thus the buckling loads achieved by the undamaged cans are at about 0.5 ± 0.1 times the prediction of the classical theory. This corresponds well to results in the literature from tests of shells having the same radius-to-thickness ratio as our specimens.

The loads reached by the damaged cans are, roughly, only half as much as those achieved by the undamaged ones. There is a lot of "scatter" in the results. Closer inspection of Fig. 4(a) shows that points corresponding to shells having smaller values of s tend to have higher ordinates than those corresponding to larger values of s; and later on we shall provide an explanation for this effect. There is a clear general tendency for the buckling load to increase with y. An obvious line of explanation, of course, springs from the idea that buckling occurs when the nominal stress level at the location of the "ding" reaches a definite value. Inspection of Fig. 2(c) shows that on this hypothesis the expected buckling load would be proportional to $1/(1+\cos y/a)$. A curve of this kind has been added to the diagram of Fig. 4(a); but although it has some resemblance to the trend of the data, it is obviously far from satisfactory in general. Thus, for example, the curve is symmetric about the axis $y = 0$; but the experimental data do not show such a feature.

Now it is often misleading to analyse numerical data from a set of experiments without reference to the qualitative behaviour of the test specimens. We have already described, in Section 4, the way in which the damaged specimens behaved under increasing load. In particular, we have noted that a small dimple, or the "ding" itself, enlarges steadily and also changes shape, as the load increases; and we have found that a buckling collapse occurs in all cases when the diamond-shaped dent reaches a width of 25 mm.

All of this suggests a new hypothesis, according to which buckling occurs when the *nominal stress at the centre of the growing dent* reaches a critical value. It is a straightforward matter to replot the data of Fig. 4(a) according to this idea. This has been done in Fig. 4(b), for which the abscissa is the circumferential distance, say z, from the principal generator to the centre of the indentation at the point of buckling: see Fig. 1(c). We have already explained in Section 4 that the dent sometimes grows away from, and sometimes towards, the other "ding". But our qualitative records reveal the pattern of events for each particular specimen, and so enable us to evaluate z.

Figure 4(b) has also been provided with a curve $F_{max} = 1.7/(1+\cos z/a)$kN. It is clear that the plot of Fig. 4(b) fits the data significantly better than that of (a), and that in particular the points for different values of s lie well together. Note also that the anomaly of data for $y < 0$ has largely been removed by replotting the data in this way.

We conclude, therefore, that in the buckling of shells damaged in this particular way, the load-limiting feature is a dent that is initiated by the zone of local damage, that grows as the load increases, and becomes unstable when it reaches a particular size. The nominal stress level in this region when buckling occurs is about $1.7/2 \times 3.6 = 0.24$ of the classical buckling stress.

6. DISCUSSION

The most obvious point to be made at this stage concerns the sharp contrast between the behaviour of *damaged* and *undamaged* thin-shell specimens under load. The damaged shells reached, on average, about one quarter of the buckling load predicted by the classical theory, while the undamaged ones reached about one half. But much more striking than this quantitative comparison between the two sets of tests is the qualitative difference. Thus, while the undamaged specimens gave no indication that a catastrophic, unstable failure was imminent, the damaged specimens gave ample warning, by virtue of the steady growth in the size of the dent as the load increased. And indeed, the rule that catastrophic failure occurs when the diamond-shaped dent has reached a unique size enables us to predict the point of instability, while a test is proceeding, with some certainty.

The present study raises a number of questions for future research. Perhaps the most important one concerns the special size of the growing dent at which the behaviour becomes unstable. What determines this critical size? In the present experiments the critical diameter of 25 mm is not far from the circumferential wavelength of the "classical" elastic buckling mode—of a "chess-board" pattern—that has equal wavelength in both longitudinal and circumferential directions: this is approximately $6.8(at)^{0.5}$ in general, and 20 mm here [see, e.g. Flügge (1973) or Calladine (1983)]. It is unlikely, of course, that the critical size is determined by purely *elastic* considerations, as it is obvious that irreversible plastic deformation is involved, at least at the "ding" end of the growing dent. And it is obvious that the "classical" theory, that considers only small peturbations from the initial cylindrical geometry, is hardly relevant to the growth of a significant dent. It should also be remarked that the diamond-shaped pattern of post-buckled deformation that is found after the sudden collapse of drinks cans—whether undamaged or damaged—under compression also shows a feature of this same length; but it is not clear that there can be a direct relation between the point at which a growing dent becomes unstable, and the post-buckling mode that is adopted by the shell after its catastrophic collapse.

We should mention here some experiments performed by Starnes (1974) on a series of thin elastic circular cylindrical shells, each of which had a small circular hole cut out at mid-height. Under increasing axial load a stable local buckle appeared on specimens containing a hole greater than a certain size. The form of this buckle (see Fig. 2 of Starnes) was something like two "diamonds" of the kind sketched in Fig. 1(c), one on either side of the hole. As the axial load was increased further, these two diamonds grew in size; and at a certain point a general collapse occurred. Although there are several major differences between the phenomena observed by Starnes and ourselves—e.g. the relatively late appearance of the "diamonds", and the absence of inelastic effects in the tests of Starnes—it may well be worthwhile to explore the implications of their common features.

It seems possible that the "critical size" at which the growing dent becomes unstable will depend not only on the radius and thickness of the shell, but also on its axial length. It would be interesting to see what would happen if the effective length of the can were to be reduced by having the ends encased in epoxy resin.

A related question concerns the critical value of the nominal stress level at which the growing dent becomes unstable. In the present experiments on a particular set of specimens, the critical stress level is about one quarter of the classical, elastic buckling stress. Since the dent involves some irreversible, plastic straining of the material, there is need for further investigation here.

Another, simpler, question concerns the size of the dimple that forms to one side of a "ding". The observed diameter of about 6 mm is close to the half-wavelength of the axi-symmetric buckling mode of the classical theory: this is approximately $1.7(at)^{0.5}$, in general, and 5 mm here. Although the two situations are, superficially, very different, it is plausible that there is a direct connection between them.

Another point for further investigation is the extent to which the "nominal" distribution of axial compressive stress around the circumference of the shell represents the actual distribution of stress in, say, the cross-section that contains a "ding". It seems most likely that there is some concentration of axial stress at the ends of the horizontal diameter of the diamond-shaped dent. Notwithstanding this, the nominal distribution of stress must be statically equivalent to the actual distribution of stress.

A final point raised by the present investigation concerns the suddenness of the buckling failure of the specimens. The undamaged cans failed abruptly, and without warning, whereas the damaged cans had a phase of stable growth of a dent before the catastrophic failure ensued. Now for some engineering purposes it is desirable to have visible, early warning of a failure, even if this is associated with smaller ultimate strength. There is scope here for investigating the deliberate introduction of "damage" of some sort in order to make the process of buckling of thin shells more benign.

REFERENCES

Arbocz, J. (1974). The effect of initial imperfections on shell stability. In *Thin Shell Structures* (eds Y. C. Fung and E. E. Sechler), Prentice-Hall, Inc., Englewood Cliffs, N.J, pp. 205–245.

Arbocz, J. and Babcock, C. D. (1969). The effect of general imperfections on the buckling of cylindrical shells. *J. Appl. Mech.* **36**, 28–38.

Arbocz, J. and Hol, J. M. A. M. (1991). Collapse of axially compressed cylindrical shells with random imperfections. *A.I.A.A. J.* **29**, 2247–2256.

Calladine, C. R. (1983). *Theory of Shell Structures*. Cambridge University Press, Cambridge, UK.

Donnell, L. H. (1934). A new theory for the buckling of thin cylinders under compression and bending. *Trans. Am. Soc. Mech. Engrs* **56**, 795–806.

Flügge, W. (1973). *Stresses in Shells* (2nd edn). Springer-Verlag, Berlin.

Kármán, T. v., Dunn, L. G. and Tsien, H-S. (1940). The influence of curvature on the buckling characteristics of structures. *J. Aeronautical Sci.* **7**, 276–289.

Koiter, W. T. (1945) (1970). The stability of elastic equilibrium. Dissertation for the degree of Doctor in the Technical Sciences at the Technische Hooge School, Delft, Amsterdam: H. F. Paris (in Dutch). English translation by E. Riks: *Technical report AFFDL-TR*-70-25, Air Force flight dynamics laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, USA.

Koiter, W. T. (1963). The effect of axisymmetric imperfections on the buckling of cylindrical shells under axial compression. *Proc. Koninklijke Nederlandse Akademie van Wetenschappen* **66** (B), 265–279.

Koiter, W. T. (1978). The influence of more-or-less localised short-wave imperfections on the buckling of circular cylindrical shells under axial compression (in a first approximation). In *Complex Analysis and its Applications* (the I.N. Vekua Anniversary Volume), Nauka Publishing House, Moscow, pp. 242–244.

Lancaster, E. R. and Palmer, S. C. (1992). Model testing of mechanically damaged pipes containing dents and gouges. *Am. Soc. Mech. Engrs* PVP **235,** 143–148.

Reddy, B. D. and Calladine, C. R. (1978). Classical buckling of a thin-walled tube subjected to bending moment and internal pressure. *Int. J. Mech. Sci.* **20,** 641–650.

Starnes, J. H. (1974). The effects of cutouts on the buckling of thin shells. In *Thin-Shell Structures* (eds Y. C. Fung and E. E. Sechler), Prentice Hall, Inc., Englewood Cliffs, N. J, pp. 289–304.

Taylor, G. I. (1942). The effect of the method of support in tests of damage to thin-walled structures by underwater explosions. Ministry of Home Security. [Reprinted in Volume III of Collected Papers (ed. G. K. Batchelor, 1963), Cambridge University Press, Cambridge, UK.]

Timoshenko, S. P. (1953). *History of Strength of Materials.* McGraw-Hill Book Company, New York.

Timoshenko, S. P. and Gere, J. M. (1961). *Theory of Elastic Stability* (2nd edn). McGraw-Hill Book Company, New York.

APPENDIX

Buckling of the shell according to the classical theory

The classical buckling stress σ_{cl} for a centrally compressed thin cylindrical shell of radius a and thickness t, made from isotropic elastic material having Young's modulus E and Poisson's ratio ν is given by [see, e.g. Timoshenko and Gere (1961), Flügge (1973), Calladine (1983)]:

$$\sigma_{cl} = E(t/a)/[3(1-\nu^2)]^{0.5}. \tag{A1}$$

For a thin shell that is loaded eccentrically, as in Fig. 2(b), it is a good approximation to assume that the (global) buckling condition is reached when the stress level on the principal generator reaches, locally, σ_{cl} as in (A1). The approximation is good because the characteristic wavelengths of the buckling mode, which are of order $(at)^{0.5}$, are small in comparison with the radius a for shells with $a/t = 350$, say: see Reddy and Calladine (1978) for a detailed analysis.

It follows that, under an axial load F that is eccentric by $a/2$, F_{cl} is given by $\sigma_{cl}/2$ multiplied by the total cross-sectional area of the tube. Hence we find

$$F_{cl} = \pi E t^2/[3(1-\nu^2)]^{0.5}. \tag{A2}$$

Putting

$$E = 210\times 10^9\ \mathrm{N\,m^{-2}} \quad \text{and} \quad \nu = 0.3 \tag{A3}$$

for steel, and $t = 95\times 10^{-6}$ m, we obtain

$$F_{cl} = 3.6\ \mathrm{kN} \tag{A4}$$

to two significant figures, which is all that is warranted here.